PHYSICAL REVIEW E **66**, 015103(R) (2002)

Adaptive Boolean networks and minority games with time-dependent capacities

Aram Galstyan and Kristina Lerman
Information Sciences Institute, University of Southern California, 4676 Admiralty Way, Marina del Rey, California 90292
(Received 17 December 2001; published 22 July 2002)

In this paper we consider a network of Boolean agents that compete for a limited resource. The agents play the so called generalized minority game where the capacity level is allowed to vary externally. We study the properties of such a system for different values of the mean connectivity K of the network, and show that the system with $K=2$ shows a high degree of coordination for relatively large variations of the capacity level.

DOI: 10.1103/PhysRevE.66.015103 PACS number(s): 02.50.Le, 05.65.+b, 87.23.Ge

Complex adaptive systems composed of agents under mutual influence have attracted considerable interest in recent years. A few examples that have been studied extensively are genetic regulatory networks [1], ecosystems [2], and financial markets [3]. These kinds of systems often display rich and complex dynamics and have been shown to possess global properties that cannot be simply deduced from the details of the microscopic behavior of individual agents.

The minority game [4] (MG) is one of the simplest examples of a complex dynamical system. It was introduced by Challet and Zhang as a simplification of Arthur's El Farol Bar attendance problem [5]. The MG consists of N agents with bounded rationality that repeatedly choose between two alternatives labeled 0 and 1 (e.g., staying at home or going to the bar). At each time step, agents who made the minority decision win. In the generalized minority game [6], the winning group is 1 (0) if the fraction of the agents who chose "1" is smaller (greater) than the capacity level $\eta, 0<\eta<1$ (for $\eta=0.5$, the game reduces to the traditional MG). Each agent uses a set of S strategies to decide its next move and reinforces strategies that would have predicted the winning group. A strategy is simply a lookup table that prescribes a binary output for all possible inputs. In the original version of the game, the input is a binary string containing the last m outcomes of the game, so the agents interact by sharing the same global signal. If the agents choose either action with probability 1/2 (the random choice game), then, on average, the number of agents choosing "1" (henceforth referred to as attendance) is $(N-1)/2$ with standard deviation $\sigma=\sqrt{N}/2$ in the limit of large N. The most interesting phenomenon of the minority model is the emergence of a coordinated phase, where the standard deviation of attendance, the volatility, becomes smaller than in the random choice game. The coordination is achieved for memory sizes for which the dimension of the reduced strategy space is comparable to the number of agents in the system [7,8], $2^m \sim N$. It was later pointed out [9] that the dynamics of the game remains mostly unchanged if one replaces the string with the actual histories with a random one, provided that all the agents act on the same signal. Analytical studies based on this simplification have revealed many interesting properties of the minority model [10,11].

In addition to the original MG, different versions of the game where the agents interact using local information only (cellular automata [12], evolving random Boolean networks [13], personal histories [14]), have been studied. In particular, it was established that coordination still arises out of local interactions, and the system as a whole achieves "better than random" performance in terms of the utilization of resources. Note that although the minority game was introduced as a toy model of the financial markets, it can serve as a general paradigm for resource allocation and load balancing in multiagent systems.

In all previous studies the capacity level has been fixed as an external parameter, so the environment in which the agents compete is stationary. In many situations, however, agents have to operate in dynamic (and in general, stochastic) environments. It is interesting to see if coordinated behavior still emerges, and to what degree agents can adapt to the changing environment. We address this problem in the present paper. Namely, we study a system of boolean agents playing a generalized minority game, assuming that the capacity level is not fixed but varies with time, $\eta(t)=\eta_0+\eta_1(t)$, where $\eta_1(t)$ is a time dependent perturbation. The framework of the interactions is based on Kauffman NK random boolean nets [1], where each agent gets its input from K other randomly chosen agents, and maps the input to a new state according to a boolean function of K variables, which is also randomly chosen and quenched throughout the dynamics of the system. The generalization we make is that agents are allowed to adapt by having more than one Boolean function, or strategy, and the use of a particular strategy is determined by an agent based on how often it predicted the winning group throughout the game. Note that this approach is very different from adaptation through evolution studied previously in the context of the minority model [13].

Our main observation is that networks with small K ($K<5$) adapt to a certain degree to the changes in the capacity level. In particular, networks with $K=2$ show a tendency towards self-organization into a phase characterized by small fluctuations, hence, an efficient utilization of the resource, even for relatively large variations in the capacity level $\eta(t)$. Note, that in the Kauffman nets with $K>2$ the dynamics of the system is chaotic with an exponentially increasing length of attractors as the system size grows, while for $K<2$ the network reaches a frozen configuration. The case $K=2$ corresponds to a phase transition in the dynamical properties of the network and is often referred as the "edge of the chaos" [1]. We would like to reiterate, however, that our system is different from a Kauffman network since the agents have an internal degree of freedom, characterized by their strategies. Specifically, our system does not necessarily have periodic attractors, while in Kauffman nets periodic attractors are guaranteed to exist due to the finite phase space and quenched rules of updating.

Let us consider a set of N boolean agents described by "spin" variables $s_i=\{0,1\}, i=1,\ldots,N$. Each agent gets its input from K other randomly chosen agents, and maps the input to a new state:

$$s_i(t+1)=F_i^j(s_{k_1}(t),s_{k_2}(t),\ldots,s_{k_K}(t)), \quad (1)$$

where $s_{k_i}, i=1,\ldots,K$ are the set of neighbors, and $F_i^j, j=1,\ldots,S$ are randomly chosen boolean functions (called strategies hereafter) used by the ith agent. For each strategy F_i^j, the agent keeps a score that monitors the performance of that strategy, adding (subtracting) a point if the strategy predicted the winning (loosing) side. Let the "attendance" $A(t)$ be the cumulative output of the system at time t, $A(t)=\Sigma_{i=1}^N s_i(t)$. Then the winning choice is "1" if $A(t)\leq N\eta(t)$, and "0" otherwise. Those in the wining group are awarded a point while the others loose one. Agents play the strategies that have predicted the winning side most often, and the ties are broken randomly.

As a global measure of optimality we consider $\delta(t)=A(t)-N\eta(t)$, that describes the deviation from the optimal resource utilization. We are primarily interested in the cumulative "waste" over a certain time window:

$$\sigma=\sqrt{\frac{1}{T}\sum_{t=t_0}^{t_0+T}\delta(t)^2}. \quad (2)$$

For $\eta_1(t)=0$ this quantity is simply the volatility as defined in the traditional minority game. We compare the performance of our system to a default random choice game, defined as follows: assume that the agents are told what is the capacity $\eta(t)$ at time t, and they choose to go to the bar with probability $\eta(t)$. In this case the main attendance will be close to $\eta(t)N$ at each time step, and the fluctuations around the mean are given by the standard deviation,

$$\sigma_0^2=N\frac{1}{T}\int_{t_0}^{t_0+T}dt'\,\eta(t')[1-\eta(t')]. \quad (3)$$

We performed intensive numerical simulations of the system described above, with the number of agents ranging from 100 to 10^4, and for network connectivity K ranging from 1 to 10. Although in our simulations we used different forms for the perturbation $\eta_1(t)$, in this paper we consider periodic perturbations only. For each K, a set of strategies was chosen for each agent randomly and independently from a pool of 2^{2^K} possible boolean functions, and was quenched throughout the game. In all simulations we used $S=2$ strategies per agent. Starting from a random initial configuration, the system evolved according to the specified rules. The duration of the simulation T_0 was determined by the particular choice of $\eta(t)$. Depending on the amplitude of the perturbation, we run the simulations for 10 to 20 periods, and usually used the data for the last two periods to determine σ.

Figure 1(a) shows a typical segment of the time series of the attendance $A(t)$ for a system of size $N=1000$, network connectivity $K=2$, and a sinusoidal perturbation $\eta_1(t)$. One can see that the system is efficient—it adapts very quickly to changes in the capacity level. The inset shows the time series of the deviation $\delta(t)$. Initially there are strong fluctuations, hence poor utilization of the resource, but after some transient time the system as a whole adapts and the strength of the fluctuations decreases. In particular, for the system sizes considered in this paper (up to $N=10^4$) σ is considerably smaller than the standard deviation σ_0 in the random choice game. This should not hold for sufficiently large N, however, since $\sigma/\sqrt{N}$ increases slowly with N (see below), while for the random choice game $\sigma_0\propto\sqrt{N}$. Note also, that the agents have information only about the winning choice, but not the capacity level. This suggests that the particular form of the perturbation may not be important as long as it meets some general criteria for smoothness.

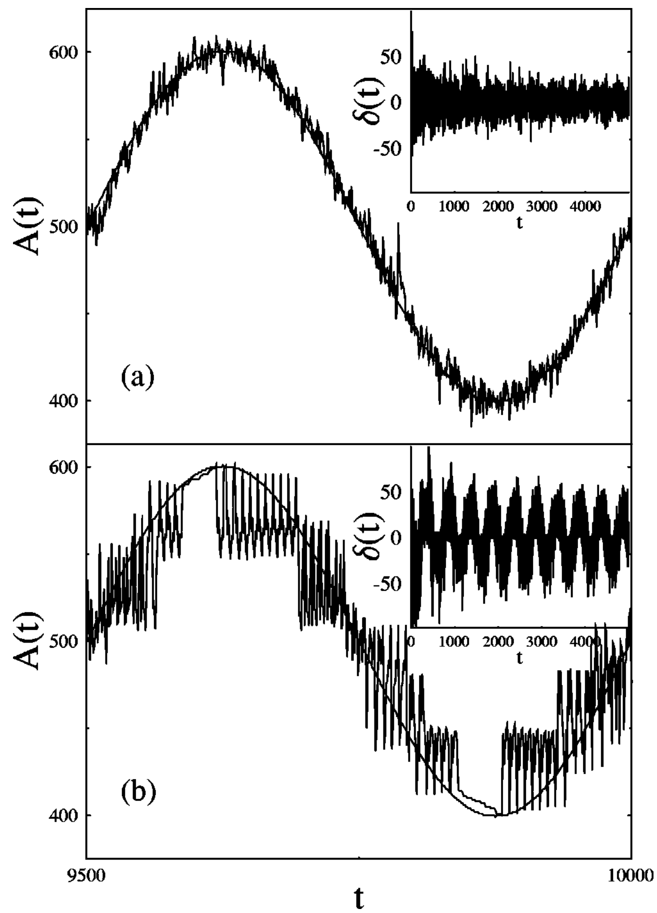

FIG. 1. A segment of the attendance time series for $\eta(t)=0.5+0.1\sin(2\pi t/T)$, $T=500$; (a) Boolean network with $K=2$, (b) traditional (generalized) minority game with $m=6$. The insets show the respective time series of the deviation $\delta(t)$.

We also studied the effect of the changing capacity level in the traditional (generalized) minority model with publicly available information about the last m outcomes of the game. We plot the attendance and deviation time series for a system with a memory size $m=6$ (corresponding to the minimum of σ) in Fig. 1(b). One can see that in this case the system also reacts to the external change; however, the overall perfor-

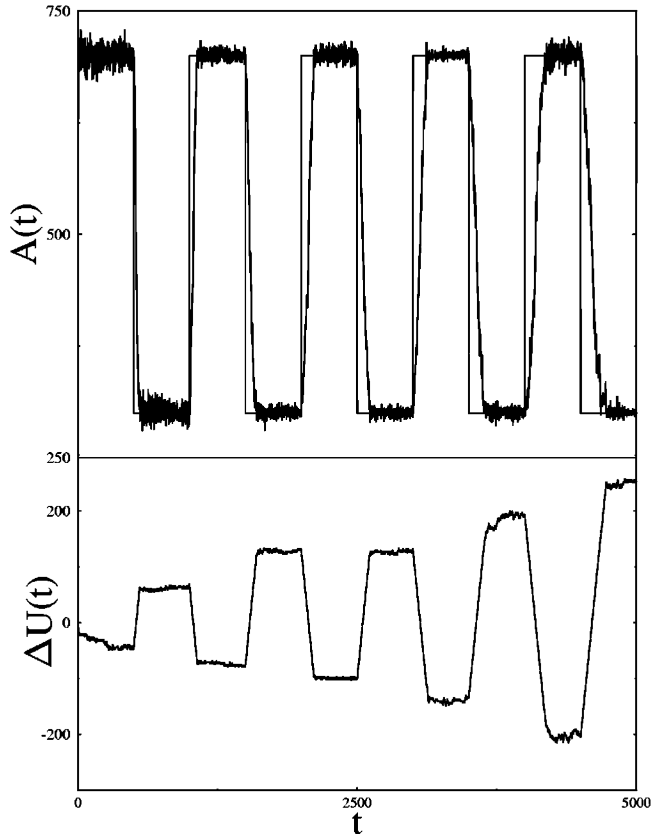

FIG. 2. Time series of attendance (top) and the gap in strategy scores (bottom) for the square-shaped capacity level variations

mance in terms of efficiency of resource allocation, as described by σ, is much poorer compared to the previous case.

Another interesting observation is that if we run the simulations long enough, the response of the system to the changing capacity level gets "out of phase" with the perturbation, leading to a gradual deterioration in the performance of the system, and the time during which the efficient phase is stable strongly depends on the rate of the changes in the capacity level, as well as on the number of agents in the system. Our results suggest that this effect is due to the increasing gap in strategy scores. As the gap in strategy scores grows, it becomes increasingly difficult for an agent to abandon a previously more successful strategy that has stopped performing well as the capacity level changes, because it takes longer for a previously loosing strategy to accumulate enough points to be played. We demonstrate this effect on a simpler, squarelike perturbation depicted in Fig. 2. One can see that each time the capacity "jumps" to its new value, it takes longer for the agents to adapt to the change. To illustrate why this happens, we plot the evolution of the gap (difference) $\Delta U(t)$ in strategy scores for one agent. For pedagogical reasons, we chose an agent with the simplest anticorrelated strategies: one of whose strategies always chooses "0" and the other "1," regardless of input. As the amplitude of the oscillations in score difference grows in time, it takes longer for the agent to switch between strategies. The same is true for the difference between strategy scores averaged over all agents, resulting in a growing lag

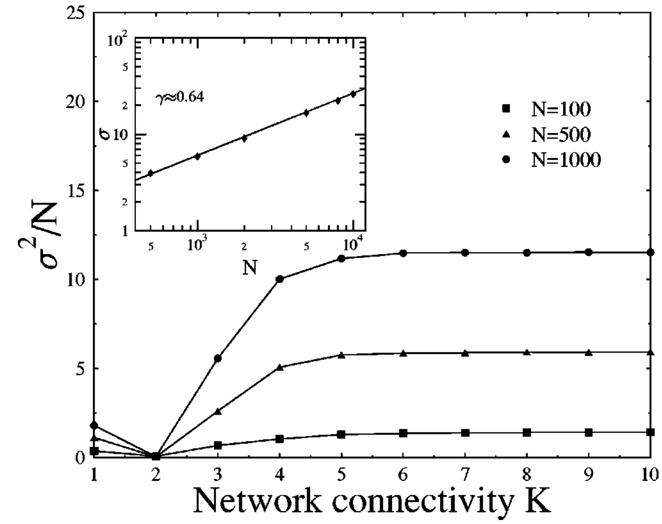

FIG. 3. σ^2/N vs the network connectivity for different system sizes and $\eta(t)=0.5+0.15\sin(2\pi t/T)$,$T=1000$. Inset plot shows the scaling relationship between σ and N for $K=2$. Average over 16 runs has been taken.

between attendance and the new capacity level after a "jump." Remarkably, one can get rid of the dephasing effect simply by introducing an upper and lower bounds for the strategy scores, thus, limiting their maximum difference.

In Fig. 3 we plot the variance per agent vs network connectivity K, for system sizes $N=100,500,1000$. For each K we performed 32 runs and averaged results. Our simulations suggest that the details of this dependence are not very sensitive to the particular form of the perturbation $\eta_1(t)$, and the general picture is the same for a wide range of functions, provided that they are smooth enough. As we already mentioned, the variance attains its minimum for $K=2$, independent of the number of agents in the system. For bigger K it saturates at a value that depends on the amplitude of the perturbation and on the number of agents in the system. We found that for large K the time series of the attendance

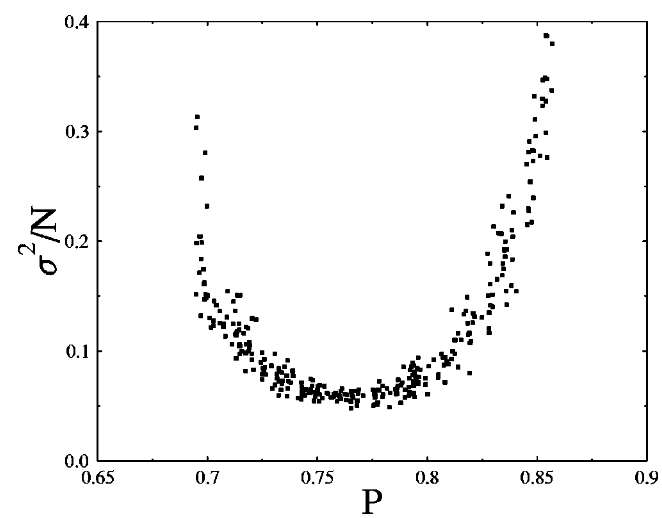

FIG. 4. Standard deviation per agent vs P for $K=3$ networks: $N=1000$, $\eta(t)=0.5+0.15\sin(2\pi t/T)$, $T=1000$.

closely resembles the time series in the absence of perturbation. This implies that for large K the agents do not "feel" the change in the capacity level. Consequently, the standard deviation increases linearly with the number of agents in the system, $\sigma \propto N$. For $K=2$, on the other hand, σ increases considerably slower with the number of agents in the system, $\sigma \propto N^{\gamma}, \gamma < 1$ (see the inset in Fig. 3). Our results indicate that the scaling (i.e., the exponent γ) is not universal and depends on the perturbation.

Though the results presented here look very interesting, we currently do not have an analytical theory for the observed emergent coordination. In contrast to the traditional minority game, where global interactions and the Markovian approximation allow one to construct a mean field description, our model seems to be analytically intractable due to the explicit emphasis on local information processing. We strongly believe, however, that the adaptability of the networks with $K=2$ is related to the peculiar properties of the corresponding Kauffman nets, and particularly, to the phase transition between the chaotic and frozen phases. It is known [15] that the phase transition in the Kauffman networks can be achieved by tuning the homogeneity parameter P that is the fraction of 1's or 0's in the output of the boolean functions (whichever is greater), with the critical value given by $P_c = 1/2 + 1/2\sqrt{1-2/K}$. To test our hypothesis, we studied the properties of networks with $K=3$ for a range of homogeneity parameter P. In Fig. 4 we plot σ^2/N vs the homogeneity parameter P. One can see that the optimal resource allocation is indeed achieved in the vicinity of the $P_c \approx 0.78$.

In conclusion, we studied a network of adaptive boolean agents competing in a dynamic environment. We established that networks with connectivity $K=2$ can be extremely adaptable and robust with respect to capacity level changes. For $K>2$ the coordination can be achieved by tuning the homogeneity parameter to its critical value. Remarkably, adaptation happens without the agents knowing the capacity level. Interestingly, the system that uses local information is much more efficient in a dynamic environment than a system that uses global information. This suggests that our model may serve as a feasible mechanism for distributed resource allocation in multiagent systems.

The authors would like to thank D. Tsigankov, J. Crutchfield, and C. Shalizi for their helpful suggestions. The research reported here was supported by the Defense Advanced Research Projects Agency (DARPA) under Contract No. F30602-00-2-0573, and in part by the National Science Foundation under Grant No. 0074790.

[1] S.A. Kauffman, *The Origins of Order* (Oxford University Press, New York, 1993).
[2] S.A. Levin, Ecosystems **1**, 431 (1998).
[3] P. W. Anderson, K. Arrow, and D. Pines, *The Economy as an Evolving Complex System* (Addison-Wesley, Redwood City, CA, 1988).
[4] D. Challet and Y.-C. Zhang, Physica A **246**, 407 (1997).
[5] W. Arthur, Am. Econ. Assoc. Papers Proc. **84**, 406 (1994).
[6] N.F. Johnson *et al.*, Physica A **269**, 493 (1999).
[7] D. Challet and Y.-C. Zhang, Physica A **256**, 514 (1998).
[8] R. Savit, R. Manuca, and R. Riolo, Phys. Rev. Lett. **82**, 2203 (1999).
[9] A. Cavagna, Phys. Rev. E **59**, R3783 (1999); A. Cavagna, Phys. Rev. Lett. **84**, 1058 (2000); R. Savit, *ibid.* **84**, 1059 (2000).
[10] D. Challet and M. Marsili, Phys. Rev. E **60**, R6271 (1999).
[11] See http://www. unifr. ch/econophysics/minority/ for an extensive collection of papers and references.
[12] T. Kalinowski, H.-J. Schulz, and M. Briese, Physica A **277**, 502 (2000).
[13] M. Paczuski, K.E. Bassler, and A. Corral, Phys. Rev. Lett. **84**, 3185 (2000).
[14] M.A.R. de Cara, O. Pla, and F. Guinea, Eur. Phys. J. B **13**, 413 (2000).
[15] B. Derrida and Y. Pomeau, Europhys. Lett. **1**, 45 (1986); **2**, 739 (1986).